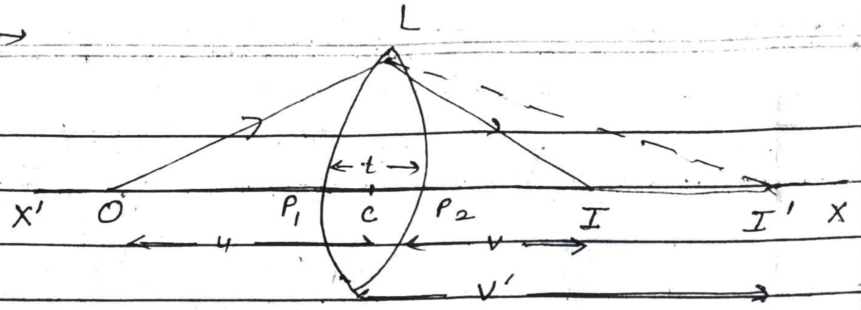
B.Sc. Part (II) Physics (Hons)

Geometrical optics

Lens - maker's formula :→



Suppose L is a thin lens. The refractive index of the material of lens is n_2 and it is placed in a medium of refractive index n_1. The optical centre of lens is C and X'X is principal axis. The radii of curvature of the surfaces of the lens are R_1 and R_2 and their poles are P_1 and P_2. The thickness of lens is t, which is very small. O is a point object on the principal axis of the lens. The distance of O from pole P_1 is $-u$. The first refracting surface forms the image of O at I' at a distance v' from P_1. From the refraction formula at spherical surface

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad \text{———} \quad (1)$$

The image I' acts as a virtual object for second surface and after refraction at second surface, the final image is formed at I. The distance of I from pole P_2 of second surface is v. The distance of virtual object (I') from pole P_2 is $(v'-t)$.

For refraction at second surface, the ray is going from second medium to first medium, therefore from refraction formula at spherical surface

$$\frac{n_1}{v} - \frac{n_2}{(v'-t)} = \frac{n_1 - n_2}{R_2} \quad \text{———} \quad (2)$$

For a thin lens 't' is negligible as compared to v'.

therefore from (2)

$$\frac{\eta_1}{v} - \frac{\eta_2}{v'} = -\frac{\eta_2-\eta_1}{R_2} \quad \text{--- ③}$$

Adding eqns (1) & (3), we get

$$\frac{\eta_1}{v} - \frac{\eta_1}{u} = (\eta_2-\eta_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\text{or, } \frac{1}{v} - \frac{1}{u} = \left(\frac{\eta_2}{\eta_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\text{i.e. } \frac{1}{v} - \frac{1}{u} = \left({}_1\eta^2-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \text{ --- ④}$$

where, ${}_1\eta^2 = \frac{\eta_2}{\eta_1}$ is refractive index of second medium (i.e. medium of lens) w.r.t. first medium.

If the object O is at infinity, the image will be formed at second focus i.e. if $u=\infty$, $v=f_2 = f$. Therefore from eqn (4)

$$\frac{1}{f} - \frac{1}{\infty} = \left({}_1\eta^2-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\text{i.e. } \frac{1}{f} = \left({}_1\eta^2-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \text{ --- ⑤}$$

If ${}_1\eta^2 = \mu$

$$\text{or, } \frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \text{ --- ⑥}$$

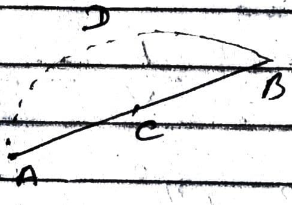
This is the formula of refraction for a thin lens.
This formula is called Lens - Maker's formula.

Fermat's Principle :-

The French mathematician Fermat, in 1958, enunciated the principle of least time for the path followed by light rays in the form :

"A ray of light in passing from one point to another through a set of media by any number of reflections or refractions chooses a ~~the~~ path along which the time taken is the least or the minimum.

It follows from Fermat's principle that the actual path followed by light between two points is the one along which the optical path is a minimum.



Hence, Fermat's principle, in general form, may be stated as :

"A ray of light in passing from one point to another through a set of media by any number of reflections or refractions chose a path along which the optical path is either a minimum or a maximum or stationary. Therefore, according to conditions of maxima & minima 't' is extremum i.e. time takes by light along the actual path is extremum.

In mathematical form Fermat's principle of extremum time may be written as

$$\int_A^B \frac{ds}{v} = \text{maximum or mimimum or stationary.} \tag{1}$$

where ds is a small element of path between any two points A and B and v is the velocity of light in the medium.

∵ Refractive Index $\mu = \frac{c}{v}$

Hence eqn (1) may be written as

$$\int_A^B \frac{\mu\, ds}{c} = \text{maximum or minimum or stationary}$$ ②

∵ speed of light (c) is always constt. therefore we may write eqn (2) in the form of Fermat's principle of extremum path as

$$\int_A^B \mu\, ds = \text{maximum or minimum or stationary}$$

i.e. $\delta \int_A^B \mu\, ds = 0$ - - - - ③

Fermat's principle is capable of deriving the following fundamental laws of Geometrical optics

(i) Rectilinear propagation of light.

(ii) laws of reflection.

(iii) laws of refraction.

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